A Mixed-Mode Simulation Technique for the Analysis of RF Circuits Driven By Modulated Signals

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ABSTRACT

This paper describes a new simulation technique of nonlinear RF circuits driven by modulated signals. Based on a true mixed-mode algorithm, it allows time-domain transient analysis of the envelope, while handling the carrier behavior in frequency-domain. Being similar in scope to the previously reported Envelope Transient Harmonic Balance [1]-[3], it is not limited, though, in excitation bandwidth.

I. INTRODUCTION

Signals handled by usual wireless telecommunication systems can be described by a high frequency RF carrier modulated by a base-band information envelope. Because this base-band envelope has a spectral content of much lower frequency than the carrier, simulating nonlinear circuits excited by this type of stimulus is a very challenging issue. In fact, we must keep in mind that, while the envelope is typically a slowly varying signal, the carrier is a very high frequency sinusoid.

The conventional time-step integration (or SPICE-like transient simulation) is thus the natural method for simulating the envelope driven circuit, as the time scale of that signal is comparable to the larger time constants involved, and the usual aperiodic stimulus' nature obviates the use of frequency-domain techniques. However, the large time constants of the bias networks determine long transient tails and, consequently, the necessity of simulating a large number of carrier periods. Time-step integration is thus inappropriate for handling the RF carrier driven circuit, and should be replaced by a frequency-domain treatment like the harmonic balance, HB, method.

The Envelope Transient Harmonic Balance technique, ETHB, [1]-[3] was precisely conceived to match these needs as it splits the simulation in two. It consists in a true mixed-mode nonlinear simulation technique that handles the envelope in time-domain and the carrier in frequency-domain. What it actually does is assuming the envelope is a so much slower varying signal than the carrier, that it can be considered constant during several carrier periods. This allows the simulator to sample the envelope in time, thus building a staircase modulated signal. This staircase stimulus version is then simulated, time-step by time-step, using a frequency-domain HB machine. The output result is finally created from all the simulated time-steps by converting the sampled envelope modulating each of the carrier harmonics back again into the continuous time domain. These are finally Fourier converted to express the result in the desired frequency-domain.

Although ETHB was initially thought to handle modulated signals, it can be applied to a very large range of stimulus, considering that the envelope can be represented as a general complex quantity. This allows the use of ETHB in situations where the input spectrum does not show a symmetry axis on the carrier (or center frequency).

Unfortunately, its authors also recognized that the basic assumption of the ETHB as also its major drawback [1]-[3]. By requiring the envelope to be a slowly varying signal compared to the center frequency, ETHB becomes restricted to circuits excited by stimulus occupying only a small fraction of the available bandwidth.

This is a serious limitation for at least three different reasons. First, as we will show in Section III, ETHB may become useless exactly where it is more needed. Second, when the bandwidth restriction is exceeded, the resulting error rises rapidly. And third, there is either no way to predict where ETHB will fail, or to evaluate the associated error of a certain simulation result.

The main goal of this paper is to show that the application of a general multi-rate partial differential equation analysis [4], [5] to the circuit allows a new ETHB formulation that is no longer limited in bandwidth.

II. THEORETICAL FORMULATION

Consider the following system of ordinary differential equations describing a general nonlinear circuit.

$$\mathbf{i}[\mathbf{y}(t)] + \frac{d\,\mathbf{q}[\mathbf{y}(t)]}{d\,t} = \mathbf{x}(t) \qquad (1)$$

$\mathbf{x}(t)$ and $\mathbf{y}(t)$ stand for the excitation and the state-variable vectors, respectively, while $\mathbf{i}[\mathbf{y}(t)]$ represents memoryless linear or nonlinear elements and $\mathbf{q}[\mathbf{y}(t)]$ models memoryless linear or nonlinear charges (capacitors) or fluxes (inductors).

In cases where $\mathbf{x}(t)$ can be assumed as a $t_e \leftrightarrow \Omega$ envelope modulating a $t_c \leftrightarrow \omega_c$ independent carrier:

$$\mathbf{x}(t) = \mathbf{X}_0 + \left[\sum_{m=-\frac{Q-1}{2}}^{\frac{Q-1}{2}} \mathbf{X}_m e^{j\Omega t} \right] \left(e^{-j\omega_c t} + e^{j\omega_c t} \right) \tag{2}$$

the nonlinear ODE of (1) can be converted into a multi-rate partial differential equation, MPDE, [4], [5] in the two independent variables t_e and t_c:

$$\mathbf{i}[\mathbf{y}(t_e,t_c)] + \frac{\partial \mathbf{q}[\mathbf{y}(t_e,t_c)]}{\partial t_e} + \frac{\partial \mathbf{q}[\mathbf{y}(t_e,t_c)]}{\partial t_c} = \mathbf{x}(t_e,t_c) \tag{3}$$

This MPDE can now be solved either directly in the time-domains (t_e,t_c) for $\mathbf{y}(t_e,t_c)$, or, alternatively, in the frequency-domains (Ω,ω_c) for the coefficients of the bi-dimensional Fourier expansions of that time state-variable vector, or even in a combination of both time- and frequency-domain. This corresponds to the case of usual modulated RF carriers by aperiodic slowly varying envelopes, where, for efficiency reasons, t_e is kept in time-domain, and t_c is transformed into the frequency-domain. Thus, the circuit can be solved in t_e using the traditional time-step integration procedure, and in ω_c by an appropriate HB algorithm. This results in the following transient envelope HB equation:

$$\mathbf{I}(t_e, k.\omega_c) + \frac{\partial \mathbf{q}[\mathbf{Y}(t_e, k.\omega_c)]}{\partial t_e} + $$
$$+\ jk.\omega_c \mathbf{Q}(t_e, k.\omega_c) = \mathbf{X}(t_e, k.\omega_c) \tag{4}$$

in which $\mathbf{I}(t_e,k.\omega_c)$, $\mathbf{Q}(t_e,k.\omega_c)$, $\mathbf{Y}(t_e,k.\omega_c)$ and $\mathbf{X}(t_e,k.\omega_c)$ are the t_e time varying Fourier components of the memoryless nonlinearities, charges or fluxes, state-variables and the excitation, respectively. The discretization of (4) using the backward Euler rule leads to the following difference equation in the above Fourier coefficients:

$$h_n.\mathbf{I}(t_{e_n}, k.\omega_c) + \mathbf{q}[\mathbf{Y}(t_{e_n}, k.\omega_c)] +$$
$$+ h_n.jk.\omega_c \mathbf{Q}(t_{e_n}, k.\omega_c) =$$
$$= h_n.\mathbf{X}(t_{e_n}, k.\omega_c) + \mathbf{q}[\mathbf{Y}(t_{e_{n-1}}, k.\omega_c)] \tag{5}$$

The proposed mixed-mode method operates by integrating (5) in a t_e time-step by time-step basis (h_n), starting from the initial conditions $\mathbf{X}(t_{e_0},k.\omega_c)$ and $\mathbf{Y}(t_{e_0},k.\omega_c)$ and solving for each of the successive time-samples t_{e_n} using the HB algorithm.

Now, we would like to point out that this MPDE formulation requires no other assumption than that t_e and t_c are independent variables, i.e., that the envelope and the RF carrier are uncorrelated signals, which is normally the case. Therefore, the above method is general and does not suffer from any bandwidth restriction as the previous ETHB. Actually, there is no fundamental reason to classify t_e as the envelope time-scale, and t_c as the high-frequency carrier time-scale. Therefore, the "envelope" could even be varying faster than de "carrier".

This is in clear opposition to the previously published ETHB, which assumed a quasi-static formulation for $\mathbf{I}(\omega)$ and $\mathbf{Q}(\omega)$, and so $\mathbf{i}(.)$ and $\mathbf{q}(.)$ are either nonlinear but algebraic or dynamic but linear. The algorithm then substitutes those frequency-domain functions by Taylor series expansions of a small number of terms around the carrier harmonics $k\omega_c$. For example, supposing that $\mathbf{I}(\omega) = \mathbf{I}(k\omega_c+\Omega)$ were Ω is a perturbation of ω_c we may have :

$$I(\omega) \approx I(k\omega_c) + \left.\frac{dI(\omega)}{d\omega}\right|_{k\omega_c} \Omega + \left.\frac{d^2 I(\omega)}{d\omega^2}\right|_{k\omega_c} \Omega^2 + ... \tag{6}$$

By using the Fourier transform rules $\Omega \leftrightarrow$ -j.$d[.]/dt_e$, $\Omega^2 \leftrightarrow -d^2[.]/dt_e^2$, etc., these Taylor expansions are then converted back into the t_e time-domain to create the envelope ordinary differential equation. The small number of terms adopted in the Taylor expansions imposes that this perturbation should be kept small, restricting the bandwidth of $\omega=k\omega_c+\Omega$ as compared to the variations of $\mathbf{I}(\omega)$ and $\mathbf{Q}(\omega)$ around ω_c.

III. APPLICATION EXAMPLE

In order to show the applicability of the method, we will now present an illustrative example. The nonlinear network represented in Fig. 1 is simply a transfer nonlinearity that drives an output parallel resonant circuit. It can be viewed as a very simplified representation of an output tuned RF amplifier.

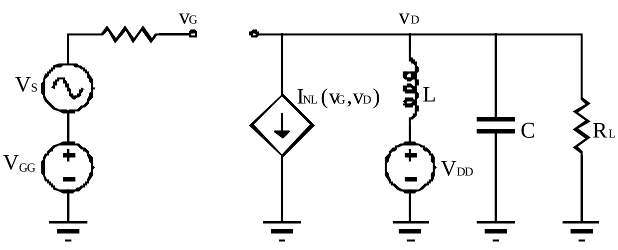

Fig. 1 – Nonlinear circuit example, for envelope transient harmonic-balance analysis.

The considered circuit excitation is the sum of a DC bias voltage (V_{GG} and V_{DD}), plus a BPSK modulated RF carrier, i.e:

$$v_s(t_e,t_c) = V_s + v_m(t_e) \left(e^{-j\omega_c t_c} + e^{j\omega_c t_c} \right) \tag{7}$$

The carrier frequency was maintained fixed at f_c=2GHz, the output band-pass filter center frequency.

The modulating signal waveform, $v_m(t_e)$, is the pseudo-random sequence shown in Fig. 2, which leads to the eye-diagram plotted in Fig. 3.

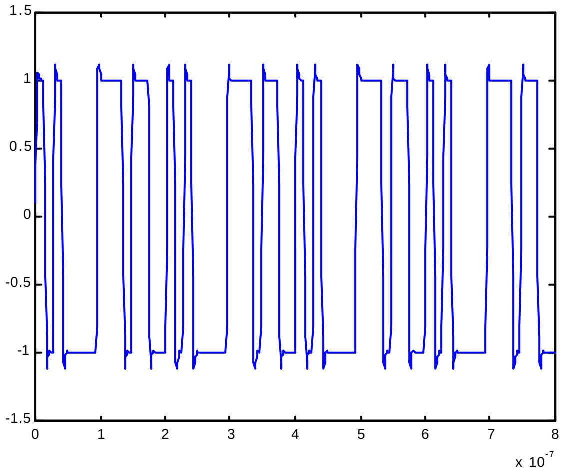

Fig. 2 – Input modulation signal waveform.

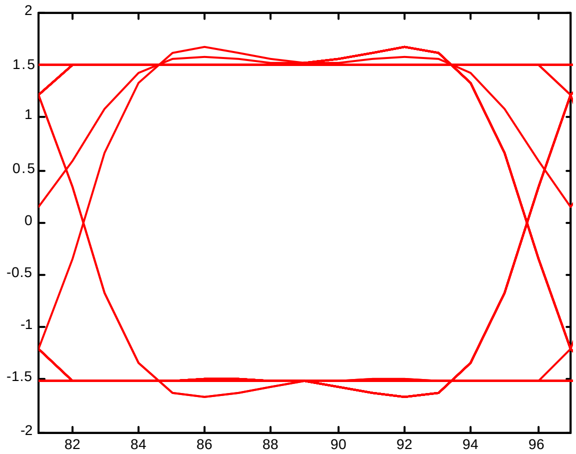

Fig. 3 – Excitation eye-diagram of the waveform shown in Fig. 2.

With this signal stimulus, two different tests, corresponding to also two distinct excitation bandwidths, were simulated.

In the first case, a bit period of 133ns was used. This corresponds to a sinusoidal fundamental modulation frequency of nearly 3.76MHz, clearly below the resonant circuit bandwidth Bw≈60MHz. Therefore, this complies with the restrictions imposed by the previous ETBH. The resulting eye-diagram is plotted in Fig. 4. As expected, it is quite similar to the input eye-diagram.

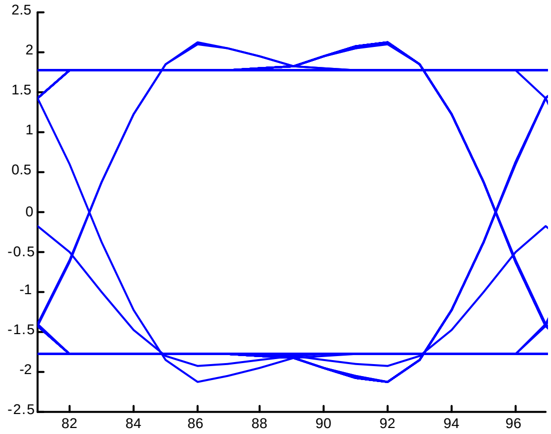

Fig. 4 – Output eye-diagram, when the envelope excitation occupies a small percentage of the circuit's bandwidth.

The second test used one tenth of the previous bit period, which locates the equivalent modulation frequency at 37.6MHz. Although this fundamental is still inside the output filter pass-band, the high frequency harmonic components of the envelope are not. The result is a significantly closed eye-diagram, as shown in Fig. 5. Therefore, it is exactly where the simulation becomes interesting that the conventional ETHB fails to produce correct results.

This illustration example shows that there may be situations of practical interest where the conventional ETHB bandwidth restrictions invalidate its use, and where the new approach is then entirely justified.

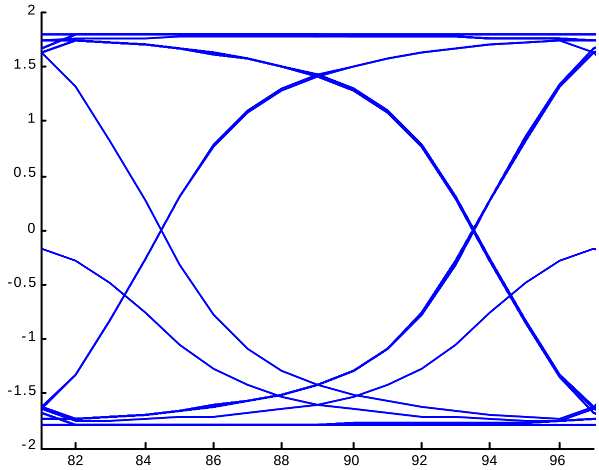

Fig. 5 – Output eye-diagram, when the envelope excitation occupies a non-negligible percentage of the circuit's bandwidth.

IV. Conclusion

A new ETBH technique founded in the recent MPDE theory was presented. Since the only restriction it poses to the excitation is that the modulating signal cannot be correlated in time with the carrier, it does not suffer from bandwidth limitations.

An illustrative circuit example was then used to prove the benefit of the present method by showing that applications of more practical interest may exactly collide with the bandwidth limitations inherent to the previous ETHB technique.

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